

CONSTRAINTS ON THE TOPOLOGY OF THE UNIVERSE DERIVED FROM THE 7-YEAR WMAP CMB DATA AND PROSPECTS OF CONSTRAINING THE TOPOLOGY USING CMB POLARISATION MAPS

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We impose constraints on the topology of the Universe determined from a search for matched circles in the cosmic microwave background (CMB) temperature anisotropy patterns of the 7-year *WMAP* data. We pay special attention to the sensitivity of the method to residual foreground contamination of the sky maps. A search for pairs of matched back-to-back circles in the higher resolution *WMAP* W-band map allows tighter constraints to be imposed on topology. Our results rule out universes with topologies that predict pairs of such circles with radii larger than $\alpha_{\min} \approx 10^\circ$. This places a lower bound on the size of the fundamental domain for a flat universe of about 27.9 Gpc. We study also the possibility for constraining the topology of the Universe by means of the matched circles statistic applied to polarised CMB anisotropy maps. The advantages of using the CMB polarisation maps in studies of the topology over simply analysing the temperature data as has been done to-date are clearly demonstrated. It is found that the noise levels of both Planck and next generation CMB experiments data are no longer prohibitive and should be low enough to enable the use of the polarisation maps for such studies.

1 Introduction

According to General Relativity, the pseudo-Riemannian manifold with signature (3,1) is a mathematical model of spacetime. The local properties of spacetime geometry are described by the Einstein gravitational field equations. However, they do not specify the global spatial geometry of the universe, i.e. its topology. This can only be constrained by observations. The concordance cosmological model assumes that the universe possesses a simply-connected topology, yet various anomalies observed on the largest angular scales in the *WMAP* data in the last decade suggest that it may be multiply-connected. Evidence of such anomalies comes from the suppression of the quadrupole moment and an alignment between the preferred axes of the quadrupole and the octopole (Copi *et al.*¹, de Oliveira-Costa *et al.*²).

We constrained the topology of the Universe (Bielewicz and Banday³) using the method of matched circles proposed by Cornish *et al.*⁴ and applied it to the 7-year *WMAP* data (Jarosik *et al.*⁵). In contrast to the majority of previous studies, we paid special attention to the impact of Galactic foreground residuals on the constraints. The method was applied to higher resolution maps than previously, which implies a lower level of false detection and therefore tighter constraints on the size of the Universe. As a result of computational limitations, we restricted the analysis to a search for back-to-back circle pairs^a.

^apairs of circles centred around antipodal points

The method of matched circles is not inherently limited to temperature anisotropy studies. It can also be applied to the CMB polarisation data (Bielewicz *et al.*⁶). We investigate also such an application of the method. We test it on simulated CMB maps for a flat universe with the topology of a 3-torus, and explicitly consider the possibility for the detection of matching circle pairs for data with an angular resolution and noise level characteristic of the Planck and CORe data. The latter is treated as a reference mission for the next generation of CMB experiments.

2 Statistic for the matched circles

If light had sufficient time to cross the fundamental cell, an observer would see multiple copies of a single astronomical object. To have the best chance of seeing ‘around the universe’ we should look for multiple images of distant objects. Searching for multiple images of the last scattering surface is then a powerful way to constrain topology. Because the surface of last scattering is a sphere centred on the observer, each copy of the observer will come with a copy of the last scattering surface, and if the copies are separated by a distance less than the diameter of the last scattering surface, then they will intersect along circles. These are visible by both copies of the observer, but from opposite sides. The two copies are really one observer so if space is sufficiently small, the CMB radiation from the last scattering surface will contain a pattern of hot and cold spots that match around the circles.

The idea of using such circles to study topology is due to Cornish *et al.*⁴. Therein, a statistical tool was developed to detect correlated circles in all sky maps of the CMB anisotropy – the circle comparison statistic

$$S_{p,r}^{\pm}(\alpha, \phi_*) = \frac{\langle 2 X_p(\pm\phi) X_r(\phi + \phi_*) \rangle}{\langle X_p(\phi)^2 + X_r(\phi)^2 \rangle}, \quad (1)$$

where $\langle \rangle = \int_0^{2\pi} d\phi$ and $X_p(\pm\phi)$, $X_r(\phi + \phi_*)$ are temperature (or polarisation) fluctuations around two circles of angular radius α centered at different points, p and r , on the sky with relative phase ϕ_* . The sign \pm depends on whether the points along both circles are ordered in a clockwise direction (phased, sign $+$) or alternately whether along one of the circles the points are ordered in an anti-clockwise direction (anti-phased, sign $-$). This allows the detection of both orientable and non-orientable topologies. For orientable topologies the matched circles have anti-phased correlations while for non-orientable topologies they have a mixture of anti-phased and phased correlations. To find correlated circles for each radius α , the maximum value $S_{\max}^{\pm}(\alpha) = \max_{p,r,\phi_*} S_{p,r}^{\pm}(\alpha, \phi_*)$ is determined. In case of anticorrelated circles the maximum value

of $-S_{p,r}^{\pm}(\alpha, \phi_*)$ is used. In the original paper by Cornish *et al.*⁴ the above statistic was applied exclusively to temperature anisotropy maps. However, it can also be applied to polarisation data and in this work, we focus on its application to the E-mode map. In this case, the X map is simply the map of the E-mode.

To draw any conclusions from an analysis based on the statistic $S_{\max}^{\pm}(\alpha)$ it is important to correctly estimate the threshold for a statistically significant match of the circle pairs. We used simulations of the maps with the same noise properties and smoothing scales as the data to establish the threshold such that fewer than 1 in 100 simulations would yield a false event.

3 Constraints on the topology of the Universe derived from 7-year WMAP data

In order to decrease the false detection level and be able to detect matched circles with smaller radius, we analyzed the *WMAP* data with the highest angular resolution i.e. the W-band map, corrected for Galactic foregrounds and smoothed with a Gaussian beam profile of the Full Width

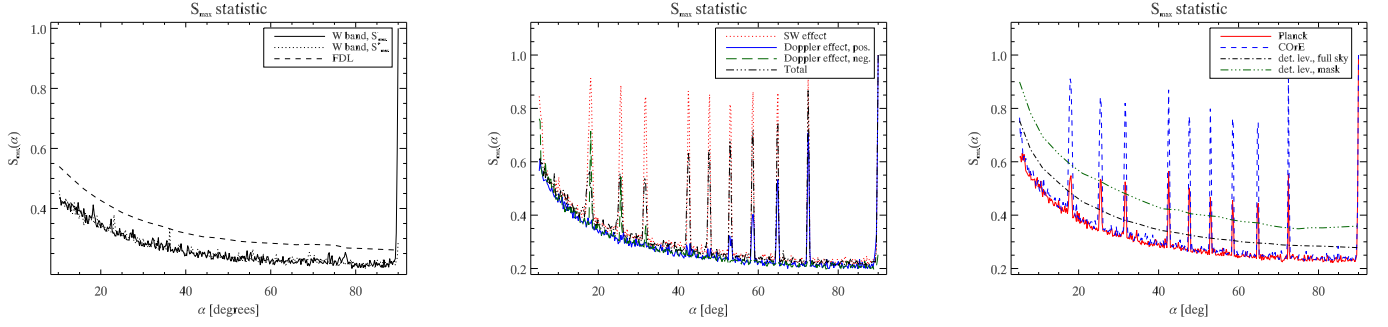


Figure 1: In the left figure, S_{\max}^{\pm} statistic for the *WMAP* 7-year W-band map. Solid and dotted lines show the statistics S_{\max}^{-} and S_{\max}^{+} , respectively, for the W-band map masked with the KQ85y7 mask. The dashed line is the false detection level estimated from 100 MC simulations. The peak at 90° corresponds to a match between two copies of the same circle of radius 90° centered around two antipodal points. In the middle and right figures, examples of the S_{\max}^{-} statistics for simulated CMB temperature and polarisation anisotropy maps, respectively, of universe with the topology of a cubic 3-torus with dimensions $L = 2 c/H_0$. In the middle figure, the dotted, solid, dashed and three dot-dashed lines show the statistic for CMB maps of the Sachs-Wolfe (SW) effect, the positive and negative correlations of the Doppler effect and total anisotropy, respectively. In the right figure, the solid and dashed lines show the statistic for simulated polarisation maps with angular resolution and noise level corresponding to the Planck and CORe data, respectively. The dot-dashed and three dot-dashed lines show the false detection levels for the statistic estimated from 100 Monte Carlo simulations of the Planck coadded 100, 143 and 217 GHz frequency polarisation maps for the full sky and cut sky analysis, respectively.

at Half Maximum (FWHM) $20'$ to decrease the noise level. The statistic for this map analysed with the KQ85y7 mask is shown in Fig. 1.

We did not find any statistically significant correlation of circle pairs in the map. As shown in Bielewicz and Banday³, the minimum radius at which the peaks expected for the matching statistic are larger than the false detection level is about $\alpha_{\min} \approx 10^{\circ}$ for the W-band map. Thus, we can exclude any topology that predicts matching pairs of back-to-back circles larger than this radius. This implies that in a flat universe described otherwise by the best-fit 7-year *WMAP* cosmological parameters, a lower bound on the size of the fundamental domain is $d = 2 R_{\text{LSS}} \cos(\alpha_{\min}) \simeq 27.9$ Gpc, where R_{LSS} is the distance to the last scattering surface. However, one has to keep in mind that this constraint concerns only those universes with such dimensions and orientation of the fundamental domain with respect to the mask that allow the detection of pairs of matched circles.

4 Prospects of constraining the topology using CMB polarisation maps

4.1 Discussion of degrading effects for temperature maps

Since the signatures of topology are imprinted on the surface of last scattering, any effects that dilute this image will also degrade the ability to detect such signatures by means of the matched circles statistic. In the case of the temperature fluctuations, there are two sources of anisotropy generated at the last scattering surface: the combination of the internal photon density fluctuations and the Sachs-Wolfe (SW) effect and the Doppler effect. In the latter case, the correlations for pairs of matched circles can be negative in universes with multi-connected topology. For pairs of back-to-back circles with a radius smaller than 45° the Doppler term becomes increasingly anticorrelated. The consequences of these degrading effects are weaker constraints on the topology of the Universe obtained from the matched circle statistic. As we can see in Fig. 1, use of the CMB map without both of the degrading effects would allow us to impose lower bounds on the minimum radius of the correlated circles which can be detected much below the present constraints $\alpha_{\min} \approx 10^{\circ}$ (Bielewicz and Banday³).

4.2 Search of matched circles in polarisation maps

A polarisation map at small angular scales can be considered as a snapshot of the last scattering surface. Theoretically, then, the polarisation provides a better opportunity for the detection of multi-connected topology signatures than a temperature anisotropy map. The only serious issues preventing its use in studies of topology are instrumental noise and the correction of the polarised data for the Galactic foreground.

The S_{\max}^- statistic for the simulated maps is shown in Fig. 1. As expected the amplitudes of the peaks do not decrease with the radius of the circles as in the case of the temperature anisotropy maps. We see that pairs of matched circles can be detected for the Planck coadded 100, 143 and 217 GHz frequency polarisation maps. However, comparing with figure for the temperature anisotropy one should notice that the relative amplitude of the peaks with respect to the average correlation level for the circles with small radius is not bigger than for the temperature map. Thus, the constraints on topology will not be much tighter than those derived from an analysis of temperature maps. A much better prospect arises for the COrE maps. The signal of the multi-connected topologies is very pronounced in this case enabling the detection of matched circles with very small radius thus providing tighter constraints on the topology of the Universe.

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^b<http://camb.info/>

^c<http://healpix.jpl.nasa.gov>